Ratios, rates, unit rates

ratios: comparison of 2 values using division

3:5 $\frac{3}{5}$ 3 to 5 Always simplify

to compare ratios with different denominators, divide.

If you divide 2 values with different units,

you end up with a unit rate (denominator = 1)

(Ex) Compare to find the better quarterback

Peyton Manning	Tom Brady
2012 Total Yards: 3812	2012 Total Yards: 3633
2012 # completions: 330	2012 # completions: 325
3812 t. yards ÷ 330 completions	3633 t. yards ÷ 325 completions
= 11.55 yards per 1 completion	= 11.18 yards per 1 completion

So according to these not-quite-exact numbers, Peyton Manning gets more yards per completion and appears to be the better QB.

Conversions: Rates use division to compare quantities, and Rates can be changed to suit ones needs.

This is where you use your green conversion sheet

(Ex) 3 miles = _____ feet

ANSWER: $3 \text{ miles} \times \frac{5280 \text{ ft}}{1 \text{ mile}} = 3 \cancel{\text{miles}} \times \frac{5280 \text{ ft}}{1 \cancel{\text{mile}}} = 15840 \text{ feet}$

(Ex) 30 mph = _____ ft/sec

ANSWER: $\frac{30 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{3600 \text{ sec}} =$ Notice how we use green facts and multiplicati

$$\frac{30 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{3600 \text{ sec}} =$$

to switch the units to go from what we had ($\frac{\text{miles}}{\text{hr.}}$) to what we want ($\frac{\text{ft}}{\text{sec}}$).

$$\frac{158400 \text{ ft}}{3600 \text{ sec}} = 44 \text{ ft/sec}$$

Proportions When 2 fractions that are equal in value are written equal to one another, it is called a proportion. $\frac{3}{4} = \frac{6}{8}$ $\frac{75}{200} = \frac{3}{8}$

If 2 fractions are proportional (equal) to one another, then 2 things are true: (1) the numerators and denominators share a multiplier, and

(2) their cross-products are equal

(Ex) Are $\frac{4}{7}$ and $\frac{12}{21}$ proportional to one another?

Multiplier option

$\frac{4}{7} \overset{\times 3}{=} \frac{12}{21}$ AND since the multiplier is 3 for the numerator AND the denominator, the 2 fractions are proportional.

Cross-product option

$\frac{4}{7} \overset{?}{=} \frac{12}{21}$

$4 \times 21 = 84$ $7 \times 12 = 84$

The fractions are proportional because their cross-products are equal.

These 2 options both allow you to find a missing value if you know the fractions are proportional.

(Ex) Find the missing value x. $\frac{10.5}{14} = \frac{12}{x}$

Multiplier option

$10.5 \times 1.14285714 = 12$

So

$14 \times 1.14285714 = 16$

You probably guessed that I divided $12 \div 10.5$ on a calculator to find this number.

Cross-multiplication option

$\frac{10.5}{14} = \frac{12}{x}$

$10.5x = 168$

therefore $x = 16$

$\frac{10.5}{} \quad \frac{10.5}{}$

$X = 16$

Sometimes questions arise that do not immediately sound/look like a proportion question. You may recognize them because they will have keywords such as "is", "of", and "percent."

When you see this, remember the proportion

$$\frac{\text{is}}{\text{of}} = \frac{\text{Percent}}{100}$$

← this 100 is a constant and is always present in these problems. It's location does not change.

(Ex) What is 32% of 195?

$$\frac{x}{195} = \frac{32}{100}$$

$X = 62.5$

(Ex) What percent of 1050 is 350?

$$\frac{350}{1050} = \frac{x}{100}$$

$X = 33.\overline{3}\ \%$

Mark-up and mark-down, percents

All of the problems in this section can be done the same way.

STEP 1: Write your percents using the percent given.

(Ex) A flat screen tv is on sale at 20% off for $1350.00. What was the original price of the TV?

20%	120%	80%

percent given | 100% + percent given | 100% − percent given

Step 2: Choose the percent appropriate for the problem.

The sale price is given to be $1350.00. A sale price is one that has had something removed from the original (100%), so the last percent (80%) makes the most sense.

Step 3: Place the percent you've picked.

$$\frac{\quad}{100\%} = \frac{80\%}{100\%}$$

← Remember this 100 is a constant so it is always there

STEP 4: Place the number into the correct location so the number matches the percent.

This number (sale price) matches the 80% sale price compared to the 100% original price. →

$$\frac{\$1350}{X} = \frac{80\%}{100\%}$$

← Remember this 100 is a constant so it is always there

If all else fails, look at what you are asked for. In this case the original price is what is unknown, so the variable goes here. ↗

Step 5: Solve the proportion

$X = \$1687.50$ original price

Review

Combine like terms, distributive property, simplify

Remember that like terms can be added or subtracted as long as they have the same variable component(s)

(Ex) Simplify $2x + 3y - 10x$

$= -8x + 3y$

Next, remember how to distribute (multiply) a coefficient into a grouping symbol.

(Ex) $4(3x + 2y - 5) + 10$

$= 12x + 8y - 20 + 10$

$= 12x + 8y - 10$

Finally, remember to always observe the order of operations. The distributive property is multiplication.

(Ex) $5 - 8(3x - 2)$

$= 5 - 24x + 16$

$= 21 - 24x$

NOTE: NOTICE THAT -8 was distributed even though the problem is "minus eight," we treat the minus as a negative.

Translate variable expressions

Review key word phrases for Addition, Subtraction, multiplication and division.

★ Pay special attention to the two special Subtraction phrases "less than" and "Subtracted from."

★ Also pay special attention to the formal phrases for the 4 operations.

ADDITION "the sum of ___ and ___"

SUBTRACTION "the difference between _____ and _____."

MULTIPLICATION "the product of _____ and _____."

DIVISION "the quotient of _____ and _____."